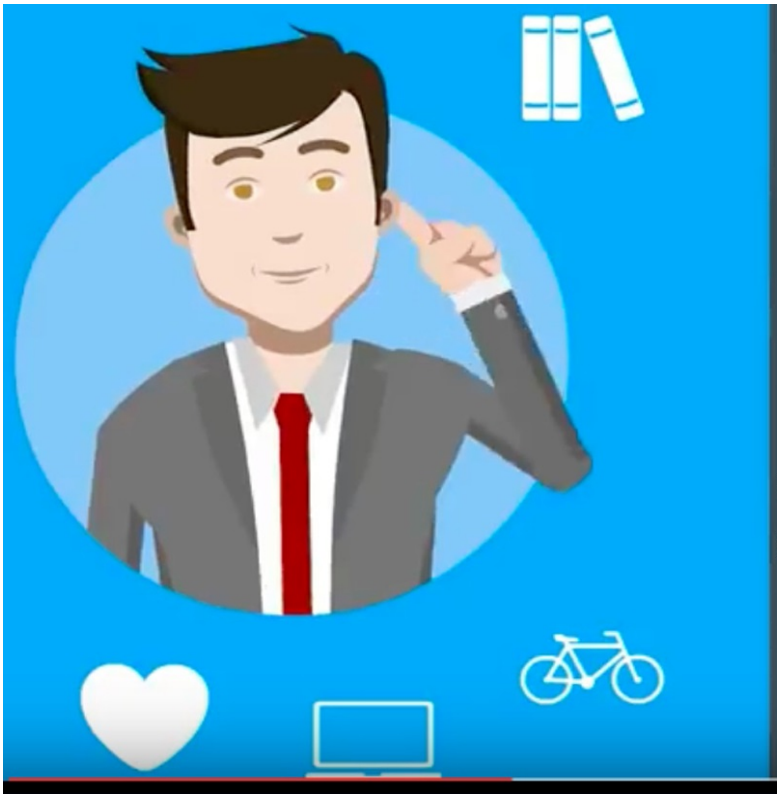

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with

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Class 13 : Analyzing polynomial equations

Lesson 7: Analyzing polynomial equations

The Factor Theorem

- If a polynomial expression has a zero (a value of x for which the polynomial equals 0) at $x = a$, it must have a factor of $(x - a)$.
- Conversely, if a polynomial has a factor of $(x - a)$, it must have a zero at $x = a$.

The function f is defined by the equation $f(x) = x^3 - ax^2 - bx + 20$ where a and b are constants. In the xy-plane, the graph of $y = f(x)$ intersects the x-axis at the points $(-2, 0)$, $(2, 0)$, and $(p, 0)$. What is the value of p?

A) 4
B) 5
C) 10
D) 20

(*Medium-hard*) Since $x = -2$ and $x = 2$ and $x = p$ are zeros of the function (that is, they are inputs that yield an output of 0), the polynomial must have $(x + 2)$, $(x - 2)$, and $(x - p)$ as factors.

$$f(x) = x^3 - ax^2 - bx + 20 = (x + 2)(x - 2)(x - p)$$

FOIL $(x + 2)(x - 2)$: $= (x^2 - 4)(x - p)$

FOIL $(x^2 - 4)(x - p)$: $= x^3 - px^2 - 4x + 4p$

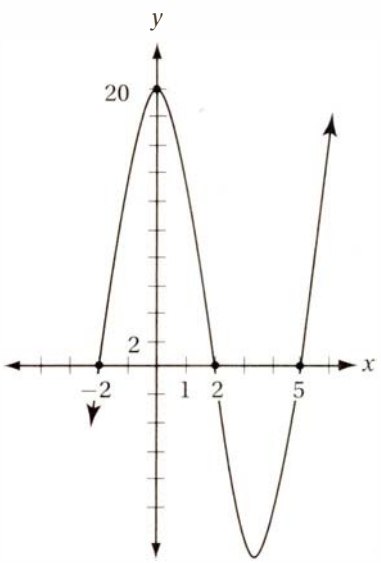
Since $x^3 - px^2 - 4x + 4p$ must be equivalent to $x^3 - ax^2 - bx + 20$, all of the corresponding coefficients must be equal. That is, $-p = -a$, $-4 = -b$, and $4p = 20$. Therefore, $p = 5$, $a = 5$, and $b = 4$, and the correct answer is (B).

Which range of values defines all of the values of x for which the function f in the previous question is positive?

A) $x < -2$ or $x > 2$
B) $-2 < x < 5$
C) $-2 < x < 2$ or $x > 5$
D) $2 < x < 5$

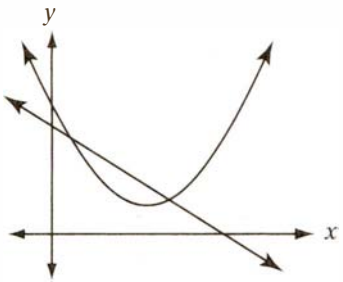
When analyzing a polynomial function, you may find it very helpful to draw its graph in the $xy =$ plane. Sometimes the x-and y-intercepts are all you need to get a good picture by hand. You should also know how to use the graphing function on your calculator, when it is permitted.

(*Hard*) This question is easier to solve if we have a graph of the function. Since we know that the equation of the function is $y = (x + 2)(x - 2)(x - 5)$, we know that it has x-intercepts at $x = -2$, $x = 2$, and $x = 5$, and a y-intercept at $y = (0 + 2)(0 - 2)(0 - 5) = 20$. Therefore, the graph looks like this:



On this graph, the points where f is positive are the points above the x-axis. This corresponds to the points where x is between -2 and 2, and where x is greater than 5. Therefore, the correct answer is (C).

Lesson 8: Systems involving quadratics



The figure above shows the graph of a system of two equations in the xy-plane. How many solutions does this system have?

A) Zero B) One C) Two D) Three

(*Easy*) Finding the solutions to a system of equations means finding the ordered pairs that satisfy all of the equations simultaneously. (If you need to review how to solve systems, see Chapter 7.) If the equations are graphed, the solutions correspond to any points where all of the graphs meet. In this case, the two graphs cross in two distinct points, so the system has two solutions and the answer is (C).

$$y + 2x = 6$$
$$y = x^2 + 3x$$

Given the system above, which of the following could be the value of y?

A) 1 or -6
B) 0 or -5
C) 0 or 10
D) 4 or 18

(*Medium*) Perhaps the simplest way to solve this system is with the process of substitution, which we applied to linear systems in Chapter 7, Lesson 12.

First equation:	$y + 2x = 6$
Substitute $y = x^2 + 3x$:	$x^2 + 3x + 2x = 6$
Subtract 6:	$x^2 + 5x - 6 = 0$
Factor with Product-Sum Method:	$(x + 6)(x - 1) = 0$
Apply Zero-Product Property:	$x = -6$ or 1

But be careful. You may be tempted to choose (A) 1 or -6, but the question asks for the value of y, not x. To find the corresponding values of y, we must plug our x-values back into one of the equations: $y = (-6)^2 + 3(-6) = 18$ or $y = (1)^2 + 3(1) = 4$; therefore, the correct answer is (D).

$$y = 1$$
$$x^2 + y^2 = 4$$
$$y = x^2$$

How many distinct ordered pairs (x, y) satisfy the three-equation system above?

A) Zero B) One C) Two D) Three

(*Medium*) To find the solutions of a system means to find the ordered pairs (x, y) that satisfy all of the equations simultaneously. Although graphing this system is not too hard, it is probably simpler to solve this system algebraically.

Substitute the first equation, $y = 1$, into the other two:	$x^2 + (1)^2 = 4$
	$1 = x^2$
Use $x^2 = 1$ to substitute into other equation:	$(1) + (1)^2 = 4$
Simplify:	$2 = 4$

Since this yields an equation that can never be true, regardless of the values of the unknowns, there is no real solution to this system, and the correct answer is (A).

If you graph this system, it will show a horizontal line, a circle, and a parabola. You will see that no point exists where all three graphs meet, indicating that the system has no solution.

Exercise Set 3 (No Calculator)

1

If $x^3 - 7x^2 + 16x - 12 = (x - a)(x - b)(x - c)$ for all values of x, what is the value of abc?

2

If $x^3 - 7x^2 + 16x - 12 = (x - a)(x - b)(x - c)$ for all values of x, what is the value of $a + b + c$?

3

1. If $x^3 - 7x^2 + 16x - 12 = (x - a)(x - b)(x - c)$ for all values of x, what is the value of $ab + bc + ac$?

4

If $x^2 - ax + 12$ has a zero at $x = 3$, what is the value of a?

5

If $x^2 - ax + 12$ has a zero at $x = 3$, at what other value of x does it have a zero?

6

$$y = 4x^2 + 2$$
$$x + y = 16$$

When the two equations in the system above are graphed in the xy-plane, they intersect in the point (a, b). If $a > 0$, what is the value of a?

7

$$x^2 + y^2 = 9$$

Which of the following equations, if graphed in the xy-plane, would intersect the graph of the equation above in exactly one point?

A) $y = -4$
B) $y = -3$
C) $y = -1$
E) $y = 0$

8

If $g(x) = a(x + 1)(x - 2)(x - 3)$ where a is a negative constant, which of the following is greatest?

A) $g(0.5)$
B) $g(1.5)$
C) $g(2.5)$
D) $g(3.5)$

9

If $2x^2 + ax + b$ has zeros at $x = 5$ and $x = -1$, what is the value of $a + b$?

A) -18
B) -9
C) -2
D) -1

10

If the graph of the equation $y = ax^4 + bx$ in the xy-plane passes through the points $(2, 12)$ and $(-2, 4)$, what is the value of $a + b$?

A) 0.5 B) 1.5 C) 2.0
D) 2.5

11

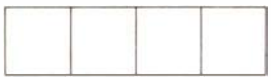
If the function $y = 3(x^2 + 1)(x^3 - 1)(x + 2)$ is graphed in the xy-plane, in how many distinct points will it intersect the x-axis?

A) Two B) Three C) Four
D) Five

Exercise Set 3 (Calculator)

12

If $x^2 + y = 10x$ and $y = 25$, what is the value of x?



13

If $2x^3 - 5x - a$ has a zero at $x = 4$, what is the value of a?

14

If $x > 0$ and $x^4 - 9x^3 - 22x^2 = 0$, what is the value of x?

15

If d is a positive constant and the graph in the xy-plane of $y = (x^2)(x^2 + x - 72)(x - d)$ has only one positive zero, what is the value of d?

16

$$y = 2x^2 + 18$$
$$y = ax$$

In the system above, a is a positive constant. When the two equations are graphed in the xy-plane, they intersect in exactly one point. What is the value of a?

17

$$4a^2 - 5b = 16$$
$$3a^2 - 5b = 7$$

Given the system of equations above, what is the value of a^2b^2?

18

For how many distinct positive integer values of n is $(n - 1)(n - 9)(n - 17)$ less than 0?

A) Six
B) Seven
C) Eight
D) Nine

19

$$x^2 + 2y^2 = 44$$
$$y^2 = x - 2$$

When the two equations above are graphed in the xy-plane, they intersect in the point (h, k). What is the value of h?

A) -8
B) -6
C) 6
D) 8

20

$$m^2 + 2n = 10$$
$$2m^2 + 2n = 14$$

Given the system of equations above, which of the following could be the value of $m + n$?

A) -7
B) -2
C) 1
D) 2

21

For how many distinct values of x does $(x^2 - 4)(x - 4)^2(x^2 + 4)$ equal 0?

A) Three
B) Four
C) Five
D) Six

22

The function $f(x)$ is defined by the equation $f(x) = a(x + 2)(x - a)(x - 8)$ where a is a constant. If $f(2.5)$ is negative, which of the following could be the value of a?

A) -2
B) 0
C) 2
D) 4

EXERCISE SET 3 ANSWER KEY

No Calculator

1. **12** When the expression $(x - a)(x - b)(x - c)$ is fully distributed and simplified, it yields the expression $x^3 - (a + b + c)x^2 + (ab + bc + ac)x - abc$. If this is equivalent to $x^3 - 7x^2 + 16x - 12$ for all values of x, then all of the corresponding coefficients must be equal.

---

2. **7** See question 1.

---

3. **16** See question 1.

---

4. **7** If $x^2 - ax + 12 = 0$ when $x = 3$, then

$(3)^2 - 3a + 12 = 0$
Simplify: $21 - 3a = 0$
Add $3a$: $21 = 3a$
Divide by 3: $7 = a$

---

5. **4** As we saw in question 4, $a = 7$.

$x^2 - 7x + 12$
Factor: $(x - 3)(x - 4)$
Therefore, the zeros are 3 and 4.

---

6. **7/4 or 1.75** $x + y = 16$
Subtract x: $y = 16 - x$
Substitute: $16 - x = 4x^2 + 2$
Subtract 16, add x: $0 = 4x^2 + x - 14$
Factor: $0 = (4x - 7)(x + 2)$
Therefore, $x = -2$ or $7/4$, but if x must be positive, it equals $7/4$.

---

7. **B** The graph of the given equation is a circle centered at the origin with a radius of 3. Therefore, the horizontal line at $y = -3$ just intersects it at $(0, -3)$. You can also substitute $y = -3$ into the original equation and verify that it gives exactly one solution.

---

8. **C** Just notice the sign of each factor for each input:
$g(0.5) = (-)(+)(-)(-) =$ negative
$g(1.5) = (-)(+)(-)(-) =$ negative
$g(2.5) = (-)(+)(+)(-) =$ positive
$g(3.5) = (-)(+)(+)(+) =$ negative
Since (C) is the only option that yields a positive value, it is the greatest.

---

9. **A** $2x^2 + ax + b$
If $x = 5$ is a zero: $2(5)^2 + 5a + b = 0$
Subtract 50: $5a + b = -50$
If $x = -1$ is a zero: $2(-1)^2 + a(-1) + b = 0$
Subtract 2: $-a + b = -2$
Multiply by -1: $a - b = 2$
Add equations: $6a = -48$
Divide by 6: $a = -8$
Substitute $a = -8$: $-8 - b = 2$
Add 8: $-b = 10$
Multiply by -1: $b = -10$
Therefore, $a + b = -8 + -10 = -18$.

---

10. **D**
Substitute (2, 12): $12 = a(2)^4 + b(2)$
Simplify: $16a + 2b = 12$
Substitute $(-2, 4)$: $4 = a(-2)^4 + b(-2)$
Simplify: $16a - 2b = 4$
Add two equations: $32a = 16$
Divide by 32: $a = ½$
Substitute: $16(1/2) + 2b = 12$
Subtract 8: $2b = 4$
Divide by 2: $b = 2$
Therefore, $a + b = 2.5$.

---

11. **A** Use the Zero Product Property. The factor $(x^2 + 1)$ cannot be zero for any value of x, $(x^3 - 1)$ is zero when $x = 1$, and $(x + 2)$ is zero when $x = -2$. Therefore, there are only two distinct points in which this graph touches the x-axis.

Calculator

12. **5** Substitute $y = 25$: $x^2 + 25 = 10x$
Subtract $10x$: $x^2 - 10x + 25 = 0$
Factor: $(x - 5)(x - 5) = 0$
Use Zero Product Property: $x = 5$

---

13. **108** If $x = 4$ is a zero: $2(4)^3 - 5(4) - a = 0$
Simplify: $108 - a = 0$
Add a: $108 = a$

---

14. **11** $x^4 - 9x^3 - 22x^2 = 0$
Divide by x^2: $x^2 - 9x - 22 = 0$
Factor: $(x - 11)(x + 2) = 0$
Use Zero Product Property: $x = 11$ or -2

---

15. **8** $y = (x^2)(x^2 + x - 72)(x - d)$
Factor: $y = (x^2)(x + 9)(x - 8)(x - d)$
By the Zero Property, the zeros are $x = 0$, -9, 8, or d. Since d is positive, but there can only be one positive zero, $d = 8$.

---

16. **12** $y = 2x^2 + 18$
Substitute $y = ax$: $ax = 2x^2 + 18$
Subtract ax: $0 = 2x^2 - ax + 18$
Divide by 2: $0 = x^2 - \frac{a}{2}x + 9$

If the graphs intersect in only one point, the system must have only one solution, so this quadratic must be a "perfect square trinomial" as discussed in Lesson 4.

$$x^2 - \frac{a}{2}x + 9 = x^2 - 2bx + b^2$$

Equate coefficients: $b^2 = 9$
$2b = a/2$
The only positive solution to this system is $b = 3$ and $a = 12$.

---

17. **144**

$4a^2 - 5b = 16$
$3a^2 - 5b = 7$
Subtract equations: $a^2 = 9$
Substitute $a^2 = 9$: $3(9) - 5b = 7$
Subtract 27: $-5b = -20$
Divide by -5: $b = 4$
Therefore, $a^2b^2 = 9(4)^2 = 144$.

---

18. **B** In order for the product of three numbers to be negative, either all three numbers must be negative or exactly one must be negative and the others positive. Since n must be a positive integer, $n - 1$ cannot be negative, and so there must be two positive factors and one negative. The only integers that yield this result are the integers from 10 to 16, inclusive, which is a total of seven integers.

---

19. **C**

$x^2 + 2y^2 = 44$
Substitute $y^2 = x - 2$: $x^2 + 2(x - 2) = 44$
Distribute: $x^2 + 2x - 4 = 44$
Subtract 44: $x^2 + 2x - 48 = 0$
Factor: $(x - 6)(x + 8) = 0$
This seems to imply that the x-coordinate of the point of intersection could be either 6 or -8, both of which are choices. Can they both be correct? No: if we substitute $x = -8$ into either equation, we get no solution, because y^2 cannot equal -8. Therefore, the correct answer is (C) 6, and the points of intersection are (6, 2) and (6, -2).

---

20. **C**

$2m^2 + 2n = 14$
$m^2 + 2n = 10$
Subtract equations: $m^2 = 4$
Take square root: $m = \pm 2$
Substitute $m^2 = 4$: $4 + 2n = 10$
Subtract 4: $2n = 6$
Divide by 2: $n = 3$
Therefore, $m + n = -2 + 3 = 1$ or $2 + 3 = 5$.

---

21. **A** Use the Zero Product Property. $(x^2 - 4)$ equals 0 if x is 2 or -2, $(x - 4)$ equals 0 if x is 4, and $(x^2 + 4)$ cannot equal 0. Therefore, there are exactly three distinct zeros.

---

22. **C**

$f(2.5) = a(2.5 + 2)(2.5 - a)(2.5 - 8)$
Simplify: $(-24.75)(a)(2.5 - a)$
This product can only be negative if a and $(2.5 - a)$ have the same sign, which is only true for (C) $a = 2$.

Polynomial factors and graphs

1. $P(x) = 2x^3 - 18x$

Given the polynomial function P defined above, what are its zeros?

A. $\{-9, -6, 2, 3\}$

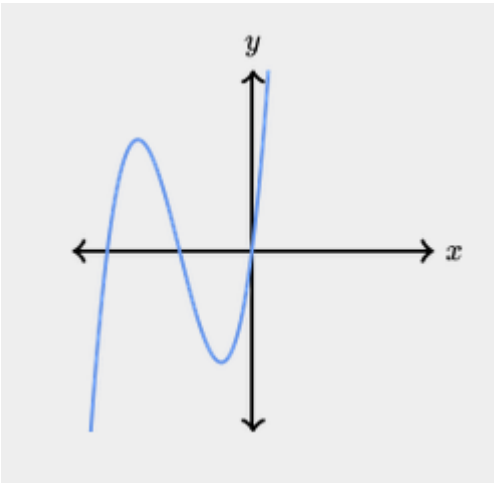
B. $\{-9, 0, 2\}$

C. $\{-3, 3\}$

D. $\{-3, 0, 3\}$

Correct Answer: D Difficult Level: 2

2. Which of the following functions could represent the graph BELOW in the xy-plane, where $y = P(x)$?



A. $P(x) = x^2 + 6x + 8$

B. $P(x) = x^3 + 6x^2 + 8x$

C. $P(x) = x^2 - 6x + 8$

D. $P(x) = x^3 - 6x^2 + 8x$

Correct Answer: B Difficult Level: 2

3. A polynomial has zeros at -9, 2, and 0. Which of the following could be the polynomial?

A. $x^2 - 7x - 18$

B. $x^3 + 7x^2 - 18x$

C. $x^3 + 7x^2 - 18x$

D. $x^3 + 6x^2 - 25x + 18$

Correct Answer: B Difficult Level: 2

4. $(x - 7)(x + 5)(2x - 3)$

Given the polynomial above, what are its zeros?

A. $\{-7, 5, -3\}$

B. $\{7, -5, 3\}$

C. $\{-7, 5, -\frac{3}{2}\}$

D. $\{7, -5, \frac{3}{2}\}$

Correct Answer: D Difficult Level: 2

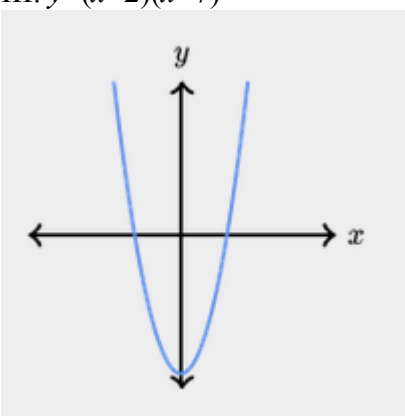
5. $2(x+55)(x-17)$
Given the polynomial above, what are its zeros?
A. $x=-55$ and $x=17$
B. $x=-55$, $x=-2$, and $x=17$
C. $x=-17$ and $x=55$
D. $x=-17$, $x=2$, and $x=55$
Correct Answer: A Difficult Level: 2

6. $G(z) = (z-1)^5-(z-1)^4$ The polynomial function G is defined above. What is the product of the zeros of G? Fill in the blank: _____

Correct Answer: 2 Difficult Level: 2

7. The polynomial function P has zeros at 3 and 6. Which of the following could be the definition of P?
A. $P(x)=x^2+9x+18$
B. $P(x)=x^2-9x+18$
C. $P(x)=x^2+3x+6$
D. $P(x)=x^2+6x+3$
Correct Answer: B Difficult Level: 2

8. Which of the following equations could represent the graph below in the xy-plane?
I. $y=(x-3)(x+3)$
II. $y=(x-4)^2$
III. $y=(x+2)(x+7)$



A. Ⅰ only
B. Ⅱ only
C. I and Ⅲ only
D. I, Ⅱ and Ⅲ
Correct Answer: A Difficult Level: 2

9. $h(t)=(t-8)^1(t-4)^2(t-2)^3(t-1)^4$

The polynomial function h is defined above. How many distinct zeros does h have?

Correct Answer: 4 Difficult Level: 2

10. $x^3+25x^2+50x-1000$

The polynomial above has $(x-5)$ and $(x+10)$ as factors. What is the remaining factor?

A. $(x+2)$

B. $(x-2)$

C. $(x+2)$

D. $(x-20)$

Correct Answer: C Difficult Level: 2

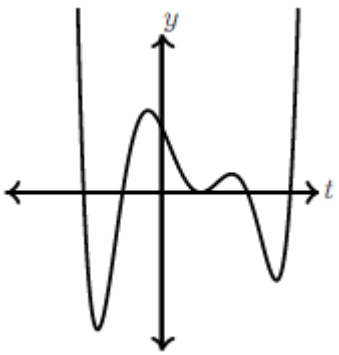
11. x^3+7x^2-36

The polynomial above has zeros at -6 and 2. If the remaining zero is z, then what is the value of $-z$?

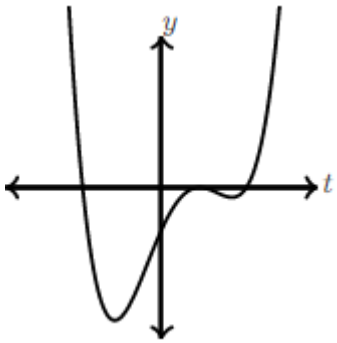
Correct Answer: 3 Difficult Level: 3

12. The function p is a polynomial of t such that $(t-10)$, $(22-t)$, $(t+10)$, and $(20+t)$ are all factors of $p(t)$. Which of the following could be the graph of $y=p(t)$ in the ty-plane?

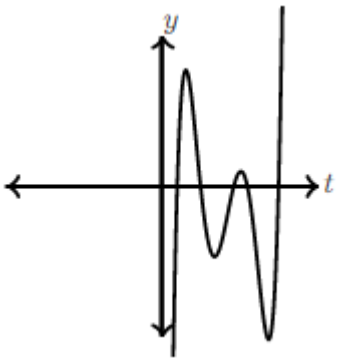
A.



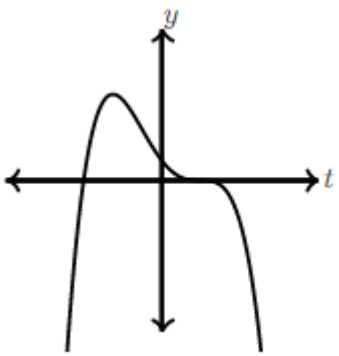
B.



C.

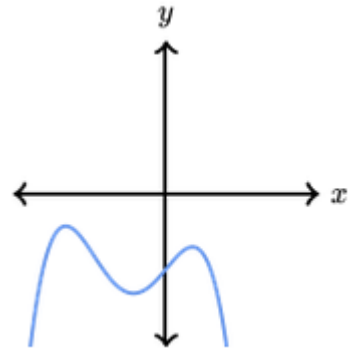


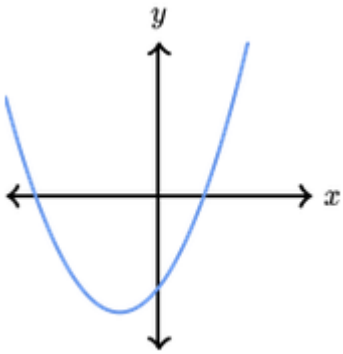
D.

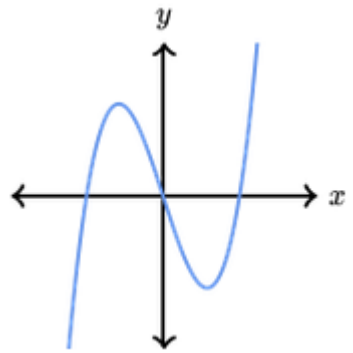


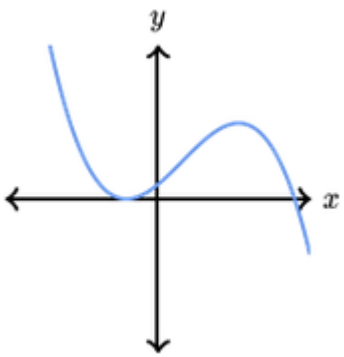
Correct Answer: A Difficult Level: 3

13. Which of the following graphs appears to represent a polynomial function with a double zero?

A. 

B. 

C. 

D. 

Correct Answer: D Difficult Level: 3

14. $g(x)=x^4-4x^3+6x^2-4x+1$

The function g is defined above. Given that all zeros of g are integers between -1 and 1inclusive, how many distinct zeros does g have? [ ]

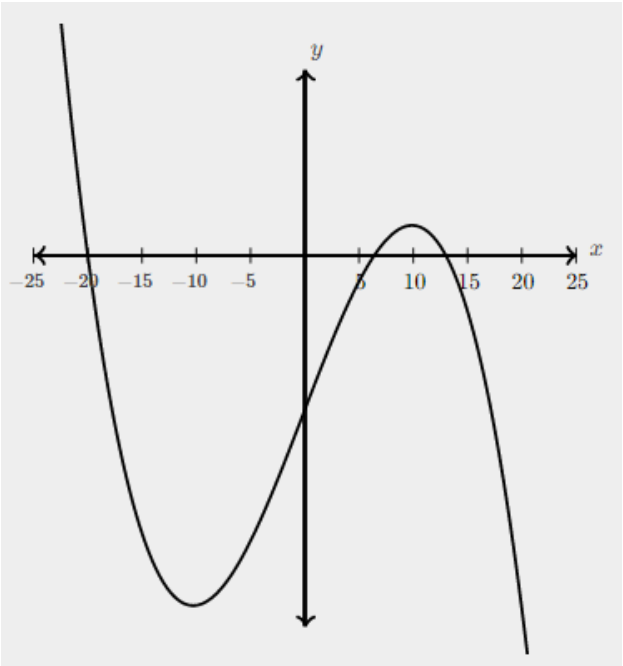
Correct Answer: [1] Difficult Level: 3

15. $x^2-ax+24$

If one of the zeros of the polynomial above is 8, what is the other zero? [ ]

Correct Answer: [3] Difficult Level: 3

16. For a function g, the graph of y=g(x)$y=g(x)$ is shown BELOW. When $g(x)$ is divided by $(x+10)$, the remainder is -20. Which of the following is closest to the remainder when $g(x)$ is divided by $(x-10)$?



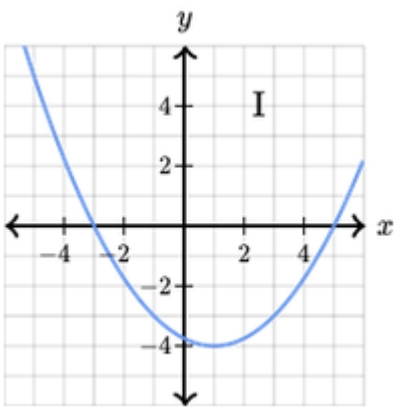
A. -28
B. -2
C. 2
D. 28
Correct Answer: C Difficult Level: 3

17. The polynomial function f is defined as $f(c)=(c-k)(c^2-4c+4)$ where k is a constant. The value 2 is a zero of f. What is the remainder of $f(c)$ when divided by $(c-2)$?

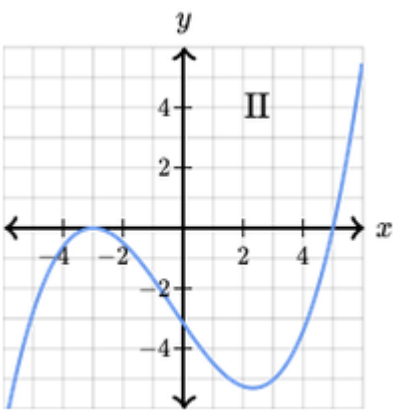
Correct Answer: 0 Difficult Level: 3

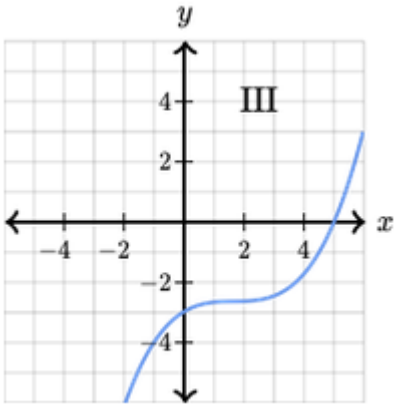
18. Which of the following graphs in the xy-plane have -3 and 5 as all of their distinct zeros for $-6 \le x \le 6$?

I.



II.



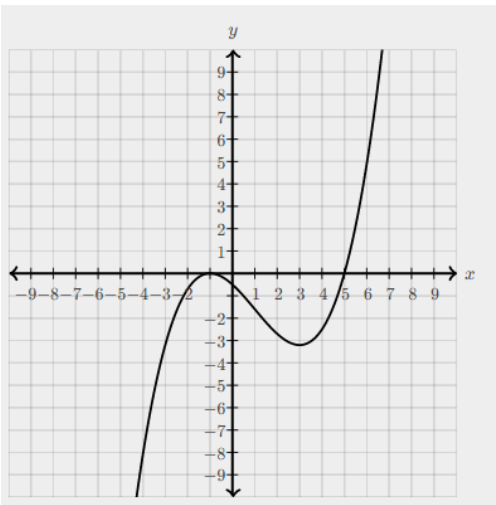


III.

A. I only
B. I and II only
C. II and III only
D. I. II and III.

Correct Answer: B Difficult Level: 3

19. Given some rational constant a, which polynomial equation could represent the graph BELOW in the xy-plane?



A. $y=a(x+1)^2(x-5)$
B. $y=a(x+1)\ (x-5)$
C. $y=a(x+1)\ (2x+1)(x-5)$
D. $y=a(x-1)\ (x+5)^2$

Correct Answer: B Difficult Level: 3

20. The equation $s=(t+3)^2(t+2)(t+1)(t)(t-1)$ is graphed on the st-plane. What is the product of the t-intercepts of the graph?

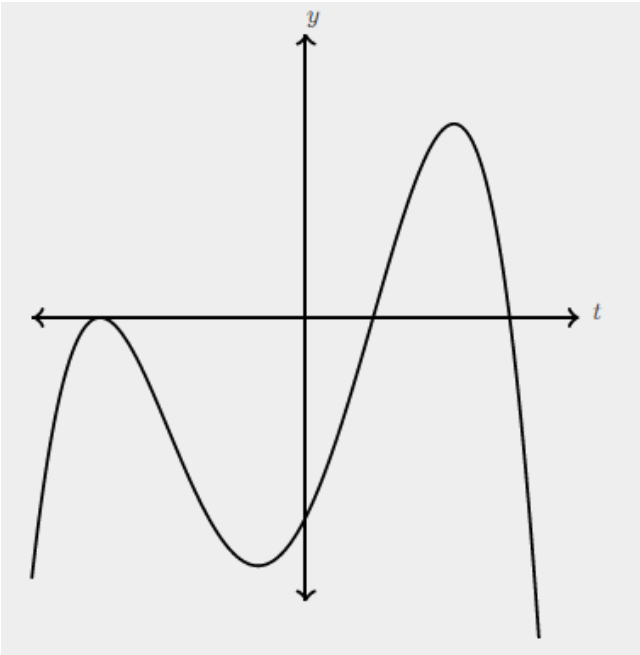
Correct Answer: 0 Difficult Level: 3

21. $q(v)=(v-8)(v-5)(v-4)(v+5)(v+10)$
The function q is defined above. If the sum of the zeros of q is s, what is the value of s?

Correct Answer: 2 Difficult Level: 3

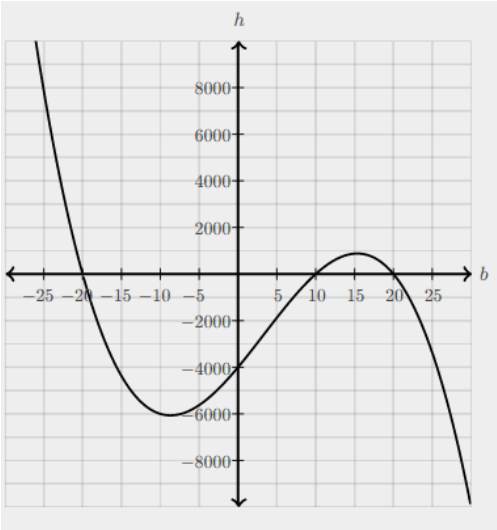
22. $(x-\sqrt{3})^2(x-\sqrt{7})$
Given the polynomial above, what are its zeros?
A. $x=-\sqrt{3}$ and $x=-\sqrt{7}$
B. $x=\sqrt{3}$ and $x=\sqrt{7}$
C. $x=3$ and $x=\sqrt{7}$
D. $x=-3$ and $x=-\sqrt{7}$
Correct Answer: B Difficult Level: 3

23. The graph of the polynomial equation $y=\alpha(t)$ is shown BELOW. Which of the following must be true?



A.The leading coefficient is positive.
B.The sum of the distinct t intercepts is negative.
C.The constant coefficient is positive.
D.The product of the distinct t intercepts is negative.
Correct Answer: D Difficult Level: 3

24. The graph shown at left could represent which of the following equations?



A. h=−(b−10)(b−20)(b+20)
B. h=(b−10)(b−20)(b+20)
C. h=−(b+10)(b−20)(b+20)
D. h=(b+10)(b−20)(b+20)
Correct Answer: A Difficult Level: 3

25. x^3+7x^2-36
The polynomial above has zeros at −6 and 2. If the remaining zero is z, then what is the value of $-z$?

Correct Answer: 3 Difficult Level: 3

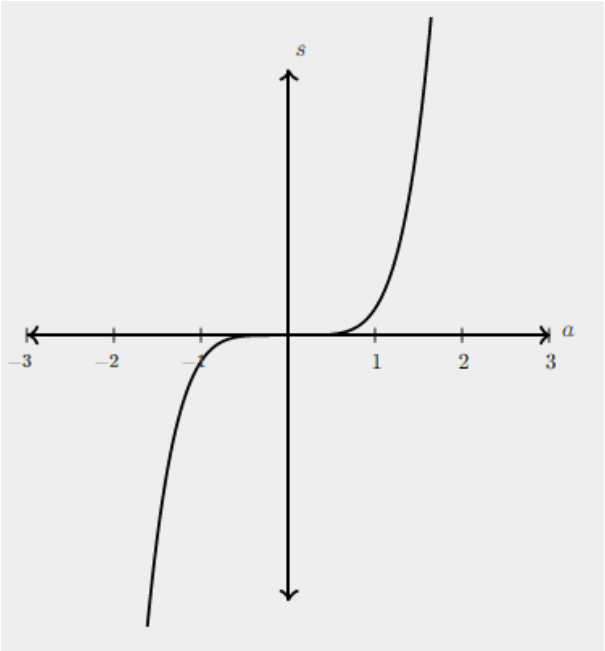
26. The polynomial function f is defined as $f(m)=(m3-m2-17m-15)(m+1)$. When $f(m)$ is divided by $(m+1)$, what is the remainder?

Correct Answer: 0 Difficult Level: 4

27. $p(n)=(n^3-12n^2+47n-60)(n-4)$
$q(n)=(n+13)(n-4)$
The functions p and q are defined above. One of the functions has a zero at $n=5$. What is $(p+q)(5)$?

Correct Answer: 18 Difficult Level: 4

28. Which of the following could be the equation corresponding to the graph BELOW?



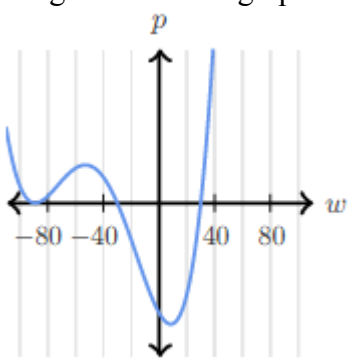
A. s=a·a·a·a
B. s=(a−1)(a−1)
C. s=a·a·a·a·a
D. s=(a−1)(a−1)(a−1)

Correct Answer: C Difficult Level: 4

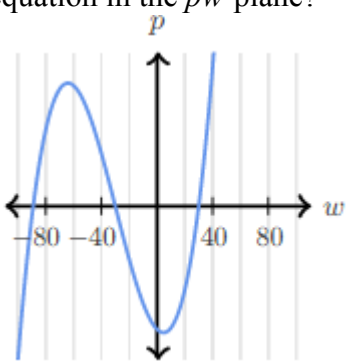
29. $p=(w-30)(w2+178w+7921)$

Given that −89 is a double zero of the polynomial equation above, which of the following could be the graph of the equation in the *pw*-plane?

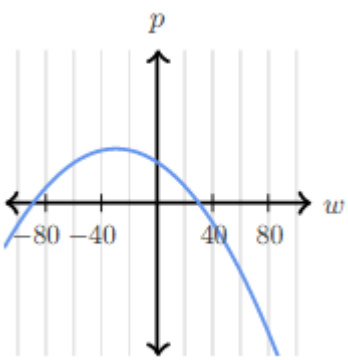
A.



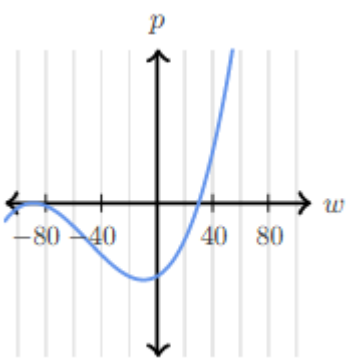
B.



C.



D.



Correct Answer: D Difficult Level: 4

30. $\ell(x)=x^4+36x^2-10{,}000$

The polynomial function ℓ is defined above. What is the remainder of $\ell(x)$ when divided by $(x+10)$?

Correct Answer: 3600 Difficult Level: 4

31. $g(w)=(w+13)^3(w+19)^2$

The polynomial function g is defined above. When $g(w)$ is divided by $(w+16)$, the remainder is r. What is the value of $|r|$?

Correct Answer: 243 Difficult Level: 4

32. A function pp is defined as $p(x) = (x-a)(x-15)(x-20)+15$ where a is a constant. Given that $p(7)=15$, what is the value of a?

Correct Answer: 7 Difficult Level: 4

33. A function w is a defined as $w(x)=ax2+bx+c$ where a, b, and c are constants. If $a=3$ and $w(3)=w(15)=0$, then what is the absolute value of b?

Correct Answer: 54 Difficult Level: 4

34. A function s is defined as $s(x)=(x-4)(x-5)^2$. A function h is defined as $h(x)=(x-a)s(x)$. For some constant a, $(x-a)^3$ is a factor of h. What is $s(a)$?

Correct Answer: 0 Difficult Level: 4

35. $12x^2+ax+2$

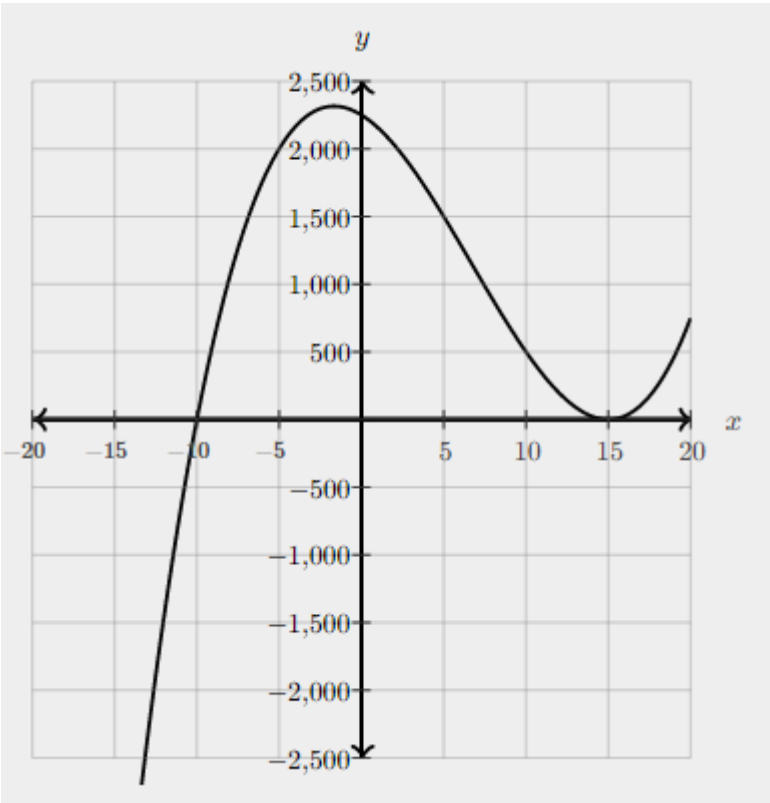
If one of the zeros of the above polynomial is $\frac{2}{3}$, what is the other zero?

Correct Answer: $\frac{1}{4}$ Difficult Level: 4

36.For any polynomial function h, the polynomial function g is defined to be $g(w)=(w-25)(w-1)h(w)$. If h has zeros at 1 and 3 only, what is the sum of the distinct zeros of g?

Correct Answer: 36 Difficult Level: 4

37. The polynomial function ff is defined as $f(x)=(x-c1)(x-c2)(x-c3)\square(x-cn)$ for some positive integer n. Each of the values $c1$, $c2$, $c3$,,, cn is a real number. The graph of $y=f(x)$ is shown BELOW. Which of the following could be the value of n?



A. 0
B. 1
C. 2
D. 3

Correct Answer: D Difficult Level: 4

38. The polynomial function f is defined as $f(m)=(m3-m2-17m-15)(m+1)$. When $f(m)$ is divided by $(m+1)$, what is the remainder?

Correct Answer: 0 Difficult Level: 4

Polynomial factors and graphs

1. $P(x) = 2x^3 - 18x$

Given the polynomial function P defined above, what are its zeros?

A. $\{-9, -6, 2, 3\}$

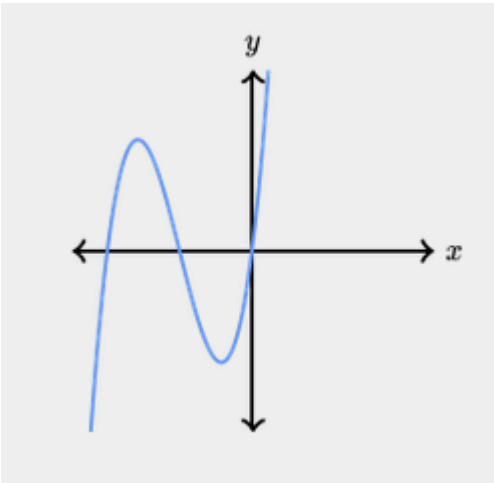
B. $\{-9, 0, 2\}$

C. $\{-3, 3\}$

D. $\{-3, 0, 3\}$

Correct Answer: D Difficult Level: 2

2. Which of the following functions could represent the graph BELOW in the xy-plane, where $y = P(x)$?



A. $P(x) = x^2 + 6x + 8$

B. $P(x) = x^3 + 6x^2 + 8x$

C. $P(x) = x^2 - 6x + 8$

D. $P(x) = x^3 - 6x^2 + 8x$

Correct Answer: B Difficult Level: 2

3. A polynomial has zeros at -9, 2, and 0. Which of the following could be the polynomial?

A. $x^2 - 7x - 18$

B. $x^3 + 7x^2 - 18x$

C. $x^3 + 7x^2 - 18x$

D. $x^3 + 6x^2 - 25x + 18$

Correct Answer: B Difficult Level: 2

4. $(x - 7)(x + 5)(2x - 3)$

Given the polynomial above, what are its zeros?

A. $\{-7, 5, -3\}$

B. $\{7, -5, 3\}$

C. $\{-7, 5, -\frac{3}{2}\}$

D. $\{7, -5, \frac{3}{2}\}$

Correct Answer: D Difficult Level: 2

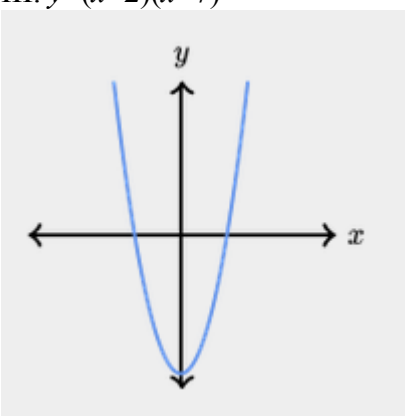
5. $2(x+55)(x-17)$
Given the polynomial above, what are its zeros?
A. $x=-55$ and $x=17$
B. $x=-55$, $x=-2$, and $x=17$
C. $x=-17$ and $x=55$
D. $x=-17$, $x=2$, and $x=55$
Correct Answer: A Difficult Level: 2

6. $G(z) = (z-1)^5-(z-1)^4$ The polynomial function G is defined above. What is the product of the zeros of G? Fill in the blank: _____

Correct Answer: 2 Difficult Level: 2

7. The polynomial function P has zeros at 3 and 6. Which of the following could be the definition of P?
A. $P(x)=x^2+9x+18$
B. $P(x)=x^2-9x+18$
C. $P(x)=x^2+3x+6$
D. $P(x)=x^2+6x+3$
Correct Answer: B Difficult Level: 2

8. Which of the following equations could represent the graph below in the xy-plane?
I. $y=(x-3)(x+3)$
II. $y=(x-4)^2$
III. $y=(x+2)(x+7)$



A. Ⅰ only
B. Ⅱ only
C. I and Ⅲ only
D. I, Ⅱ and Ⅲ
Correct Answer: A Difficult Level: 2

9. $h(t)=(t-8)^1(t-4)^2(t-2)^3(t-1)^4$

The polynomial function h is defined above. How many distinct zeros does h have?

Correct Answer: 4 Difficult Level: 2

10. $x^3+25x^2+50x-1000$

The polynomial above has $(x-5)$ and $(x+10)$ as factors. What is the remaining factor?

A. $(x+2)$

B. $(x-2)$

C. $(x+2)$

D. $(x-20)$

Correct Answer: C Difficult Level: 2

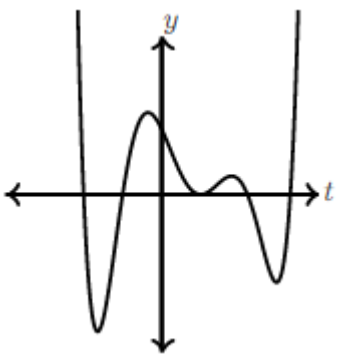
11. x^3+7x^2-36

The polynomial above has zeros at -6 and 2. If the remaining zero is z, then what is the value of $-z$?

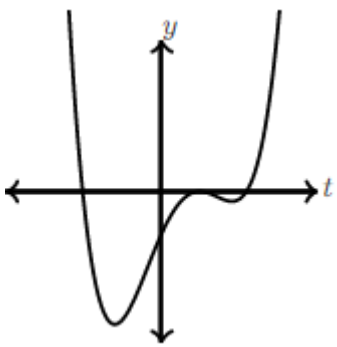
Correct Answer: 3 Difficult Level: 3

12. The function p is a polynomial of t such that $(t-10)$, $(22-t)$, $(t+10)$, and $(20+t)$ are all factors of $p(t)$. Which of the following could be the graph of $y=p(t)$ in the ty-plane?

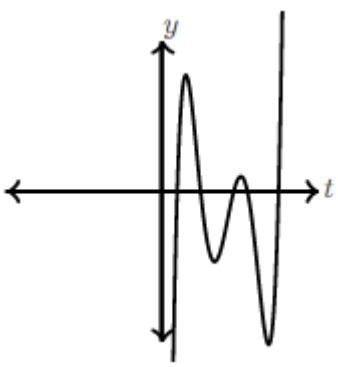
A.



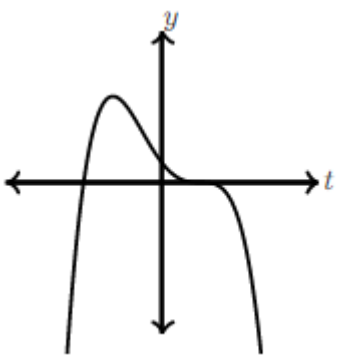
B.



C.

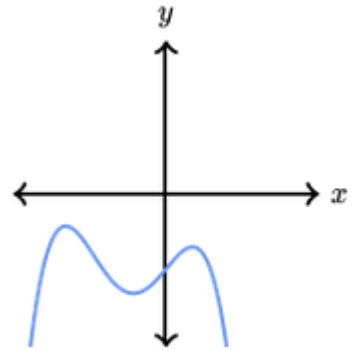


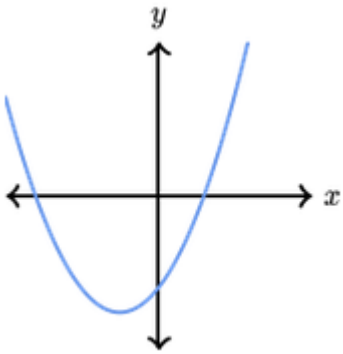
D.

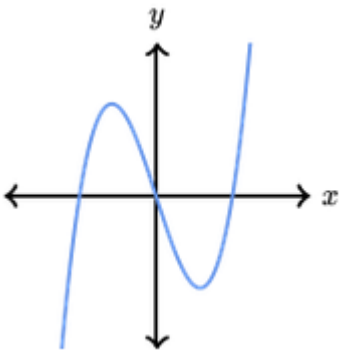


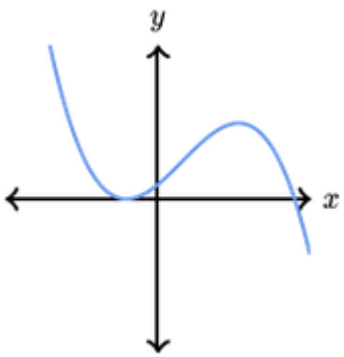
Correct Answer: A Difficult Level: 3

13. Which of the following graphs appears to represent a polynomial function with a double zero?

A. 

B. 

C. 

D. 

Correct Answer: D Difficult Level: 3

14. $g(x)=x^4-4x^3+6x^2-4x+1$

The function g is defined above. Given that all zeros of g are integers between -1 and 1inclusive, how many distinct zeros does g have? [ ]

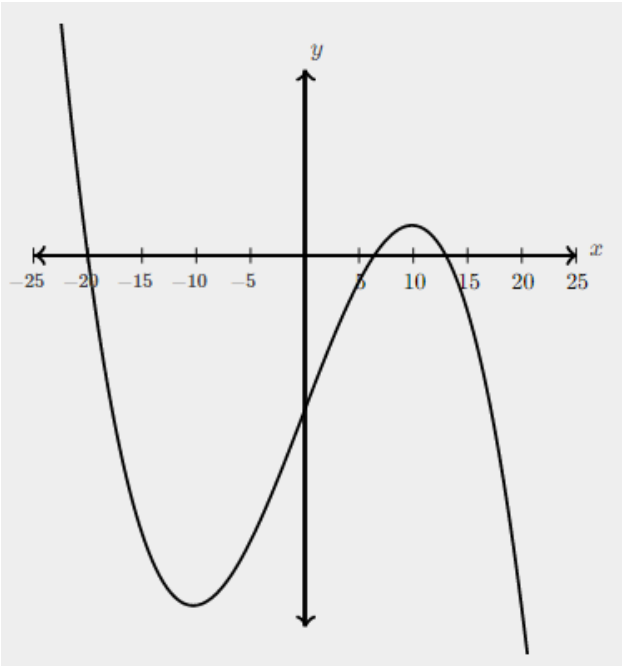
Correct Answer: [1] Difficult Level: 3

15. $x^2-ax+24$

If one of the zeros of the polynomial above is 8, what is the other zero? [ ]

Correct Answer: [3] Difficult Level: 3

16. For a function g, the graph of y=g(x)$y=g(x)$ is shown BELOW. When $g(x)$ is divided by $(x+10)$, the remainder is -20. Which of the following is closest to the remainder when $g(x)$ is divided by $(x-10)$?



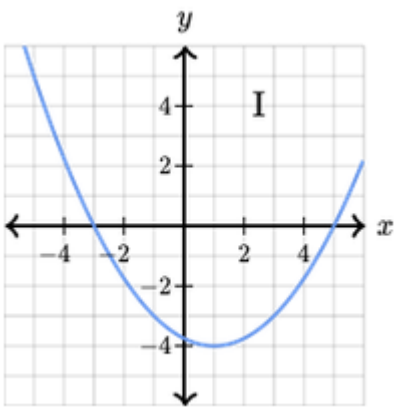
A. -28
B. -2
C. 2
D. 28
Correct Answer: C Difficult Level: 3

17. The polynomial function f is defined as $f(c)=(c-k)(c^2-4c+4)$ where k is a constant. The value 2 is a zero of f. What is the remainder of $f(c)$ when divided by $(c-2)$?

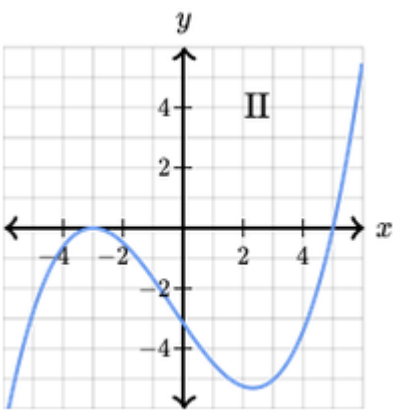
Correct Answer: 0 Difficult Level: 3

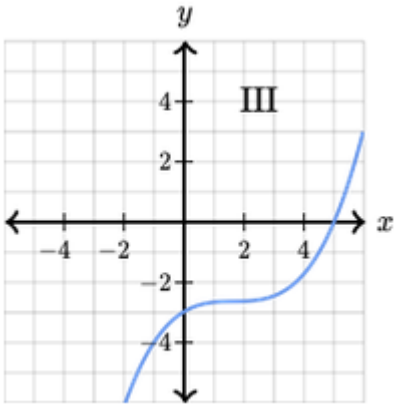
18. Which of the following graphs in the xy-plane have -3 and 5 as all of their distinct zeros for $-6 \le x \le 6$?

I.



II.



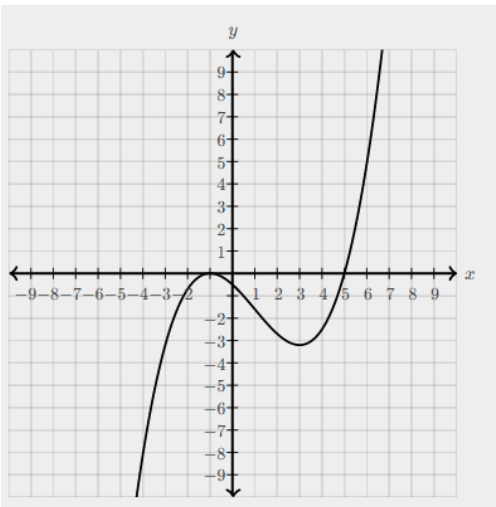


III.

A. I only
B. I and II only
C. II and III only
D. I. II and III.

Correct Answer: B Difficult Level: 3

19. Given some rational constant a, which polynomial equation could represent the graph BELOW in the xy-plane?



A. $y=a(x+1)^2(x-5)$
B. $y=a(x+1)\ (x-5)$
C. $y=a(x+1)\ (2x+1)(x-5)$
D. $y=a(x-1)\ (x+5)^2$

Correct Answer: B Difficult Level: 3

20. The equation $s=(t+3)^2(t+2)(t+1)(t)(t-1)$ is graphed on the st-plane. What is the product of the t-intercepts of the graph?

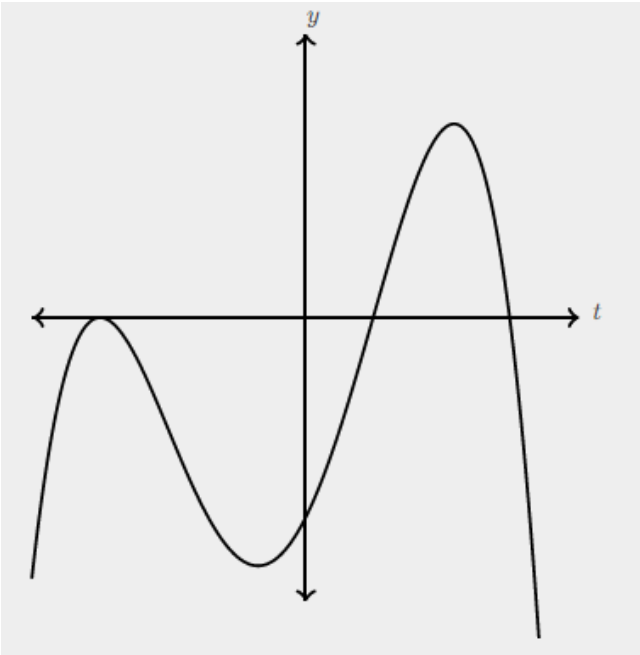
Correct Answer: 0 Difficult Level: 3

21. $q(v)=(v-8)(v-5)(v-4)(v+5)(v+10)$
The function q is defined above. If the sum of the zeros of q is s, what is the value of s?

Correct Answer: 2 Difficult Level: 3

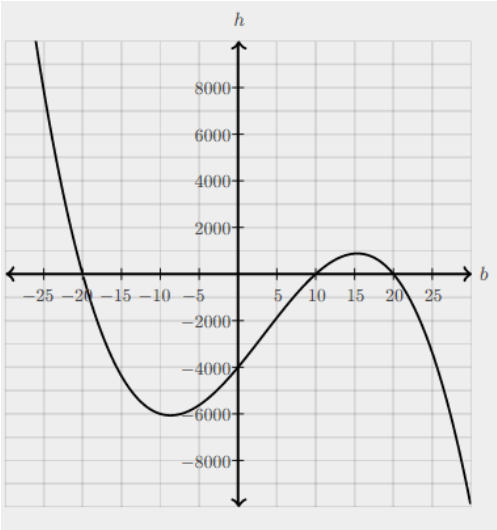
22. $(x-\sqrt{3})^2(x-\sqrt{7})$
Given the polynomial above, what are its zeros?
A. $x=-\sqrt{3}$ and $x=-\sqrt{7}$
B. $x=\sqrt{3}$ and $x=\sqrt{7}$
C. $x=3$ and $x=\sqrt{7}$
D. $x=-3$ and $x=-\sqrt{7}$
Correct Answer: B Difficult Level: 3

23. The graph of the polynomial equation $y=\alpha(t)$ is shown BELOW. Which of the following must be true?



A.The leading coefficient is positive.
B.The sum of the distinct t intercepts is negative.
C.The constant coefficient is positive.
D.The product of the distinct t intercepts is negative.
Correct Answer: D Difficult Level: 3

24. The graph shown at left could represent which of the following equations?



A. h=−(b−10)(b−20)(b+20)
B. h=(b−10)(b−20)(b+20)
C. h=−(b+10)(b−20)(b+20)
D. h=(b+10)(b−20)(b+20)
Correct Answer: A Difficult Level: 3

25. x^3+7x^2-36
The polynomial above has zeros at −6 and 2. If the remaining zero is z, then what is the value of $-z$?

Correct Answer: 3 Difficult Level: 3

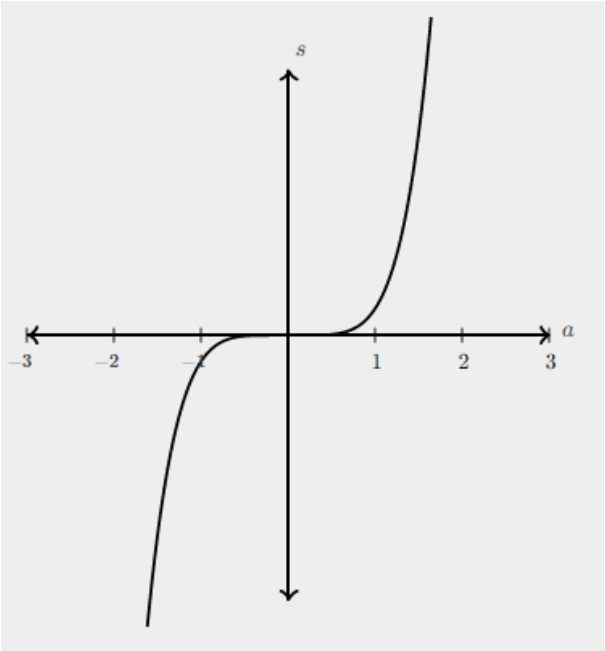
26. The polynomial function f is defined as $f(m)=(m3-m2-17m-15)(m+1)$. When $f(m)$ is divided by $(m+1)$, what is the remainder?

Correct Answer: 0 Difficult Level: 4

27. $p(n)=(n^3-12n^2+47n-60)(n-4)$
$q(n)=(n+13)(n-4)$
The functions p and q are defined above. One of the functions has a zero at $n=5$. What is $(p+q)(5)$?

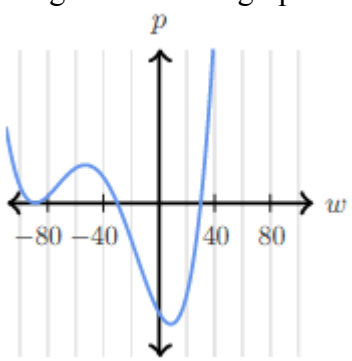
Correct Answer: 18 Difficult Level: 4

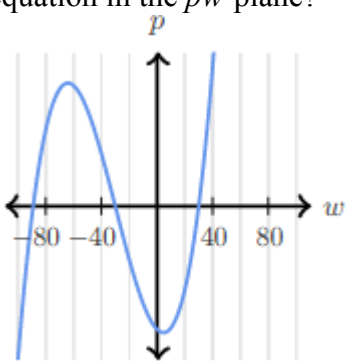
28. Which of the following could be the equation corresponding to the graph BELOW?

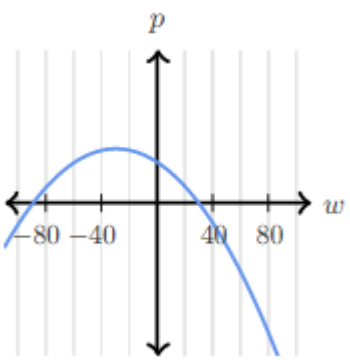


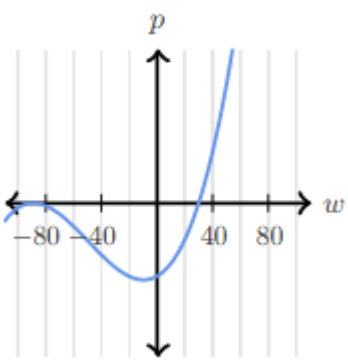
A. s=a·a·a·a
B. s=(a−1)(a−1)
C. s=a·a·a·a·a
D. s=(a−1)(a−1)(a−1)
Correct Answer: C Difficult Level: 4

29. $p=(w-30)(w2+178w+7921)$
Given that −89 is a double zero of the polynomial equation above, which of the following could be the graph of the equation in the *pw*-plane?

A.


B.


C.


D.


Correct Answer: D Difficult Level: 4

30. $\ell(x)=x^4+36x^2-10{,}000$

The polynomial function ℓ is defined above. What is the remainder of $\ell(x)$ when divided by $(x+10)$?

Correct Answer: 3600 Difficult Level: 4

31. $g(w)=(w+13)^3(w+19)^2$

The polynomial function g is defined above. When $g(w)$ is divided by $(w+16)$, the remainder is r. What is the value of $|r|$?

Correct Answer: 243 Difficult Level: 4

32. A function pp is defined as $p(x) = (x-a)(x-15)(x-20)+15$ where a is a constant. Given that $p(7)=15$, what is the value of a?

Correct Answer: 7 Difficult Level: 4

33. A function w is a defined as $w(x)=ax2+bx+c$ where a, b, and c are constants. If $a=3$ and $w(3)=w(15)=0$, then what is the absolute value of b?

Correct Answer: 54 Difficult Level: 4

34. A function s is defined as $s(x)=(x-4)(x-5)^2$. A function h is defined as $h(x)=(x-a)s(x)$. For some constant a, $(x-a)^3$ is a factor of h. What is $s(a)$?

Correct Answer: 0 Difficult Level: 4

35. $12x^2+ax+2$

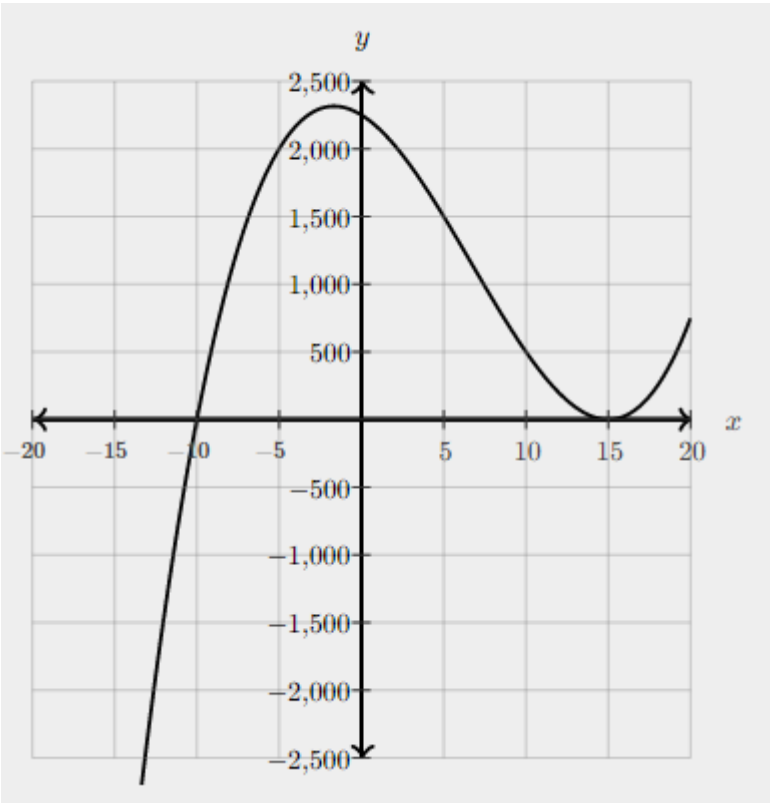
If one of the zeros of the above polynomial is $\frac{2}{3}$, what is the other zero?

Correct Answer: $\frac{1}{4}$ Difficult Level: 4

36.For any polynomial function h, the polynomial function g is defined to be $g(w)=(w-25)(w-1)h(w)$. If h has zeros at 1 and 3 only, what is the sum of the distinct zeros of g?

Correct Answer: 36 Difficult Level: 4

37. The polynomial function ff is defined as $f(x)=(x-c1)(x-c2)(x-c3)\square(x-cn)$ for some positive integer n. Each of the values $c1$, $c2$, $c3$,,, cn is a real number. The graph of $y=f(x)$ is shown BELOW. Which of the following could be the value of n?



A. 0
B. 1
C. 2
D. 3

Correct Answer: D Difficult Level: 4

38. The polynomial function f is defined as $f(m)=(m3-m2-17m-15)(m+1)$. When $f(m)$ is divided by $(m+1)$, what is the remainder?

Correct Answer: 0 Difficult Level: 4